Solutions to Test 1

1. See 1.1 in Assignment 2.

2. For all $x \in \mathbb{R}^n$,

$$(f * g)(x) = \int_{\mathbb{R}^n} f(x-y)g(y)\,dy = \int_{\mathbb{R}^n} e^{-|x-y|^2} dy.$$

Let $z = x - y$. Then

$$(f * g)(x) = \int_{\mathbb{R}^n} e^{-|z|^2} dz = \prod_{j=1}^{n} \int_{-\infty}^{\infty} e^{-x_j^2} dx_j = \pi^{n/2}.$$

3. See the second part of 3.5 in Assignment 2.

4. See 4.3 in Assignment 2.