**Solutions to Test 1**

1. By the chain rule,

$$\begin{aligned}\frac{\partial z}{\partial s} &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial s} \\ &= (e^x \sin\, y)t^2 + (e^x \cos\, y)(2st) \\ &= t^2 e^{st^2} \sin\,(s^2 t) + 2ste^{st^2} \cos\,(s^2 t).\end{aligned}$$

Also,

$$\begin{aligned}\frac{\partial z}{\partial t} = &= \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t} \\ &= (e^x \sin\, y)(2st) + (e^x \cos\, y)(s^2) \\ &= 2ste^{st^2} \sin\,(s^2 t) + s^2 e^{st^2} \cos\,(s^2 t).\end{aligned}$$

2. Let $f(x, y, z) = x^2 + y^2 + z^2$. Then

$$\frac{\partial f}{\partial x}(2, 1, 2) = 4, \frac{\partial f}{\partial y}(2, 1, 2) = 2, \frac{\partial f}{\partial z}(2, 1, 2) = 4.$$

Therefore the tangent plane to the surface at the point (2,1,2) has an equation of the form

$$\frac{\partial f}{\partial x}(2, 1, 2)(x - 2) + \frac{\partial f}{\partial y}(2, 1, 2)(y - 1) + \frac{\partial f}{\partial z}(2, 1, 2)(z - 2) = 0,$$

which is the same as

$$4(x - 2) + 2(y - 1) + 4(z - 2) = 0$$

or

$$2x + y + 2z = 9.$$

The normal line to the surface at the point (2,1,2) has an equation of the form

$$x = 2 + 4t, y = 1 + 2t, z = 2 + 4t,$$

for all $t \in (-\infty, \infty)$.

3. (**DO NOT DEDUCT ANY MARKS IF ONLY ONE DIRECTION IS GIVEN.)**
Since

$$(D_{\mathbf{u}}f)(2,0) = -(D_{-\mathbf{u}}f)(2,0),$$

it follows that $|(D_{\mathbf{u}}f)(2,0)|$ is maximum when $\mathbf{u}$ is in the same or opposite direction of $(\nabla f)(2,0)$. But

$$(\nabla f)(x,y) = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right) = (e^y, xe^y).$$

So, $(\nabla f)(2,0) = (1,2)$. Therefore $|(\nabla f)(2,0)| = \sqrt{5}$. So, $|(D_{\mathbf{u}}f)(2,0)|$ is maximum when

$$\mathbf{u} = \frac{1}{\sqrt{5}}\mathbf{i} + \frac{2}{\sqrt{5}}\mathbf{j}$$

or

$$\mathbf{u} = -\frac{1}{\sqrt{5}}\mathbf{i} - \frac{2}{\sqrt{5}}\mathbf{j}.$$

The maximum value is $|(\nabla f)(2,0)| = \sqrt{5}$.

4.(i) Since $f_x = 2x$ and $f_y = 2y$, the only critical point is $(0,0)$. Now,

$$f_{xx} = 2, f_{xy} = f_{yx} = 0, f_{yy} = 2.$$

So,

$$D = \det\begin{pmatrix} f_{xx} & f_{xy} \\ f_{yx} & f_{yy} \end{pmatrix} = \det\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 4.$$

Since $D > 0$ and $f_{xx}(0,0) > 0$, it follows that $f(0,0) = 0$ is a local minimum.
(ii) The boundary points are $(x,y)$ with $x^2 + y^2 = 4$. At each of these boundary points $(x,y)$, $f(x,y) = 4$. So $f(x,y) = 4$ is the absolute maximum at every point in $\{(x,y) : x^2 + y^2 = 4\}$ and $f(0,0)$ is the absolute minimum.