

**Mathematics 3271 3.0**

**Fall 2018**

**Assignment 4**

Due Wednesday, November 14, 2018

1. Determine whether or not each of the following functions  $f$  defined on  $[-\pi, \pi]$  is in the  $L^2$ -Sobolev space  $H^{1,2}[-\pi, \pi]$  of order 1. Explain your answers.

- $f(x) = |x|, \quad x \in [-\pi, \pi].$

- $f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0, \\ 1, & x = 0. \end{cases}$

- $f(x) = \begin{cases} \frac{\cos x}{|x|}, & x \neq 0, \\ 1, & x = 0. \end{cases}$

2. Let  $f$  be the function on  $[-\pi, \pi]$  given by

$$f(x) = e^{|x|}, \quad x \in [-\pi, \pi].$$

Find an upper bound for

$$\sup_{x \in [-\pi, \pi]} |f(x) - s_j(x)|, \quad j = 0, 1, 2,$$

where  $s_0$ ,  $s_1$  and  $s_2$  are, respectively, the first, second and third partial sums of the Fourier series of  $f$ . (Round your answers to four decimal places.)

3. Section 22 in Weinberger: 3