Mathematics 3271 3.0
Fall 2017
Assignment 3

1. Prove that a Hölder continuous function f of order $\alpha > 1$ on $[a, b]$ has to be a constant function on $[a, b]$.

2. Prove that the function f defined by $f(x) = |x|$ for all x in $[-\pi, \pi]$ is Hölder continuous of order 1.

3. Let f be the function on $[-\pi, \pi]$ defined by

$$f(x) = \begin{cases} \frac{\sin x}{|x|^\alpha}, & x \neq 0, \\ 0, & x = 0, \end{cases}$$

where $\alpha \in (0, 1]$. Does the Fourier series of f converge to $f(x)$ for all $x \in [-\pi, \pi]$? If yes, give a proof. If no, explain why not.

4. Section 19 in Weinberger: 2, 5 (You do not have to compare the graphs of $f(x)$ and $s_2(x)$.)
5. Section 20 in Weinberger: 4